

Orbit Design and Optimization Based on Global Telecommunication Performance Metrics^{1,2}

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Abstract—The orbit selection of telecommunications orbiters is one of the critical design processes and should be guided by mission-specific performance metrics and constraints. In order to aid the orbit selection process, we have coupled the Telecom Orbit Analysis and Simulation Tool (TOAST) with genetic optimization algorithms. As a demonstration, we have applied the developed tool to find the optimal orbit for Mars orbiters with the constraint of traveling on a frozen orbit and the optimization goal of minimizing the telecommunication gap time. For the measurement of the gap time, several relevant metrics are constructed: 1) area-weighted average gap time, 2) global maximum of local maximum gap time, 3) global maximum of local minimum gap time. Optimal solutions are found for each type of metrics. Common features and differences between the optimal solutions as well as the advantages and disadvantages of each metric are presented. The optimal solutions are compared with several candidate orbits that were considered during the development of the Mars Telecommunications Orbiter mission.

Tool (TOAST) with genetic optimization algorithms.

In a previous study, Lee et al. showed that TOAST provides global telecom performance metrics of an orbiter with a variety of central planetary bodies and a range of orbiter design parameters such as orbit elements, transmitter power, antenna gain, and frequency band [2]. In the current study, genetic optimization algorithms are coupled to TOAST in order to efficiently explore the design space and to solve the optimization problems. As an initial study, we have applied the developed tool to select an optimal orbit for Mars relay orbiters.

The remainder of the paper is organized as follows. The optimization metrics and constraints that are relevant to the orbit selection process are discussed in Sec. 2. The reason we chose the genetic algorithm as the optimization method is presented in Sec. 3. Our optimization results are presented and discussed in Sec. 4. Finally, we summarize our findings in Sec. 5.

TABLE OF CONTENTS

1. INTRODUCTION	1
2. TELECOM METRICS AND CONSTRAINTS	1
3. WHY GENETIC OPTIMIZATION ALGORITHMS	2
4. OPTIMAL ORBIT SELECTION.....	2
5. CONCLUSIONS	6
ACKNOWLEDGEMENTS	8
REFERENCES.....	8
BIOGRAPHY.....	8

1. INTRODUCTION

Observational orbiters such as the Mars Global Surveyor, the Mars Odyssey, and the Mars Express have not only advanced human understanding of Mars through detailed observations but have also served as a communications relay station for other missions [1]. For such a multifunctional orbiter, telecommunication performance is one of the metrics to consider in the process of orbit selection. In order to aid the orbit selection process, we have coupled the Telecom Orbit Analysis and Simulation

2. TELECOM METRICS AND CONSTRAINTS

Telecommunication Metrics

For the orbit selection problem based on telecommunication performance, typical optimization goals are 1) to maximize data volume that the orbiter can process and 2) to minimize the gap time between communications. For each of the two goals, several relevant performance metrics can be constructed. For the goal of maximizing data volume, possible metrics are 1) maximum instantaneous data rates, 2) average data volumes over all communications opportunities, 3) number of contacts per planet's day (Sol), 4) cumulative contact hours per Sol. In the spirit of minimizing the gap time, reasonable metrics are 1) global maximum of local maximum gap time, 2) global maximum of local minimum gap time, 3) area-weighted gap time. It is not obvious whether all the performance metrics will yield a common optimal orbit or a different one. Some of the metrics may compete in such a way that improving one metric will degrade other metrics.

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Telecommunication Constraints

In addition to the telecom metrics, the orbit selection problem involves constraints such as being a sun-synchronous, daily ground-track repeating, and frozen orbit. Each constraint results in the reduction of the feasible orbit design space.

The sun-synchronous orbit means that the orbit plane maintains a fixed geometry with respect to the planet-Sun line. In other words, the rate of change of the ascending node is equal to the orbital rate of the planet around the Sun. This type of orbit provides constant Sun angles relative to an orbiter, which is viewing a fixed location on the planet's surface. The sun-synchronous feature is useful since it saves spacecraft power and simplifies spacecraft thermal control. The constraint of making the orbit sun-synchronous imposes an equality condition given by a function of the eccentricity and the inclination.

Similarly, the daily ground-track repeating feature can be required to allow for contacts with every point on the surface at the same local time. The daily ground-track repeating constraint requires the orbiter's orbit period to be commensurable to the planet's inertial rotation period.

The frozen orbit feature is characterized by keeping the argument of the perigee and the eccentricity of the orbit constant. The regularity of the frozen orbit ensures that, for a given latitude, the orbiter always passes at the same altitude. The frozen-orbit constraint restricts the inclination angle to be either 63.4 or 116.6 degrees for an elliptical orbit is when the planet oblateness effect is included. The angles are known as critical inclinations.

When perturbation forces such as third-body gravitational forces, planet oblateness effect, or atmospheric drag are included, the constraint becomes more complicated and may require occasional propulsive maneuvers to adjust the orbit. Meeting all the constraints may be impossible in some cases (i.e. over-conditioned problem) or may result in degraded performance metrics. Therefore, one should always carefully consider the cost of adding new constraints to the performance metrics.

3. WHY GENETIC OPTIMIZATION ALGORITHMS

For this orbit selection and optimization process, a genetic optimization algorithm is chosen for its simplicity and flexibility compared to traditional mathematical programming techniques such as gradient-descent methods, conjugate direction methods, and nonlinear programming. The genetic algorithm is inspired by natural selection and the sexual reproduction process of living organisms [3,4]. The algorithm implements abstracted biological processes including stochastic mutation, recombination, and selection to explore the design space and is shown to be efficient in finding a global optimal solution in a high-dimensional, multimodal, rugged, and constrained search space.

The reason why the genetic algorithm is more suitable particularly for the orbit selection problem than traditional methods is three-fold. First, the objective function given by telecom metrics is often rugged, whence it follows that gradient-based methods are not applicable. An abrupt change of the gradient misguides the search direction in the traditional gradient-based algorithms. The genetic algorithm does not depend on the gradient and is therefore more robust in a rugged search space [3,4].

Second, the variables to optimize in the orbit selection problem can be a collection of various types such as integer values, real values, and options rather than one type of variables. A traditional method is typically designed to handle one type of variables. For example, integer programming is for integer variables while nonlinear programming is for real variables. The genetic algorithm is flexible enough to handle various types directly or indirectly by encoding them into binary genes [3,4].

Third, the objective function in the orbit selection process is complicated and typically ill-defined. The objective function can be a combination of various telecom metrics with different constraints associated to each metric. The overall optimization goal can involve multiple competing objectives rather than a single one. The optimization priority order of the multiple objectives is often unknown until later mission design phases. For such an ill-defined objective function, the traditional method is either not applicable or yields a single point solution that becomes meaningless when the objective function is redefined. The genetic algorithm is less vulnerable to ill-defined objective functions. It has a mechanism to take into account the error and noise in the objective function and to generate k-best solutions rather than a single point solution if needed. Furthermore, the genetic algorithm can optimize multiple objectives simultaneously without introducing an arbitrary priority order or weighting factors [5,6].

4. OPTIMAL ORBIT SELECTION

Problem Setup – Frozen Orbit Optimization

We have applied the developed tool to select an optimal orbit for general Mars orbiters with the constraint of being a frozen orbit and with the optimization goal of minimizing the time between communications (i.e. gap time). A list of variables in this optimization includes semimajor axis, eccentricity, inclination, and argument of perigee. Since only one orbiter is considered, the time of perigee and the argument of the ascending node are irrelevant variables for the single orbiter's telecommunication performance. Due to the frozen-orbit constraint, the inclination angles for elliptical orbits (non-zero eccentricity) are restricted to two values (63.4 and 116.6 degrees) while any inclination angle is allowed for a circular orbit.

Gravitational Model

Our gravitational model for the central planet, Mars in this case, includes the planet oblateness effect represented by the J_2 term in the geopotential expansion. Higher order geopotential terms and the gravitational forces of the Sun and Martian moons are not included.

Performance Metrics

Three metrics relevant to the gap-time measurement are considered in this orbit selection: 1) area-weighted gap time (Area-Average), 2) global maximum of local maximum gap time (Max-Max), 3) global maximum of local minimum gap time (Max-Min). Mathematically, the metrics are given by

$$\begin{aligned} \text{Area - Average Gap} &= \frac{1}{N} \sum_{i,j} t(\theta_i, \phi_j) \cos(\theta_i), \\ \text{Max - Max Gap} &= \max_i \left[\max_j [t(\theta_i, \phi_j)] \right], \\ \text{Max - Min Gap} &= \max_i \left[\min_j [t(\theta_i, \phi_j)] \right], \end{aligned} \quad (1)$$

where $t(\theta_i, \phi_j)$ is the maximum communication gap time of the surface grid centered at latitude θ_i and longitude ϕ_j . And, N is the number of simulation grids on the planet's surface.

Planet Surface Grid Resolution

The three performance metrics are based on the global map of the telecommunication gap times and thus depends on the resolution for the global maps of the planet's surface. The convergence of the gap times with respect to the resolution of the global map is monitored, as plotted in Figure 1. As the granularity of the longitude and latitude of the global map improves from 30 degrees to 1 degree, the gap times converges. We choose 5 degrees as the resolution of the global map for the optimization process, as it properly represents the telecommunication metrics.

Simulation Time

A simulation time should be long enough for the telecommunication performance metrics to converge. Especially when the orbit period is incommensurable to the planet's internal rotation period, a ground-track repeating time is very long or even infinite. As a result, a simulation time required to reasonably estimate the performance metric is also very long. The metric convergence is examined with respect to the simulation time. Figure 2 shows the convergence of the metrics for various semimajor axes. Except for the semimajor axis of 4000 km, the Area-Average gap converges gradually, the Max-Max gap converges quickly after a few Sols, and the Max-Min gap jumps abruptly once and converges thereafter. The orbit with the semimajor axis of 4000 km

has a ground area that is continuously missed which leads to the increase of the gap time as the simulation time increases. The missed area is near the poles due to the low altitude and the low inclination (15 degrees) of the orbit.

Overall, the simulation time of 15 Martian Sols is long enough to obtain an approximate gap time if the gap time converges. If the gap time increases linearly with the simulation time, it indicates that the gap time either diverges or converges to a much larger gap time than other converged solutions. Therefore, the simulation time of 15 Martian Sols is still reasonable to approximately estimate the gap times in comparison with better solutions.

Genetic Algorithm Parameter Setting

For the genetic algorithm, the following parameters are used. The population size is 100, the crossover probability is 0.8, the mutation probability per gene/variable is 1/4, the elitist fraction in the population is 0.2 (meaning that the best 20% solutions are passed to the next generation/iteration without mutation and crossover), and the maximum number of generations is 40. The algorithm parameters are empirically chosen for an optimal performance for a reasonable computational time. The variables are encoded into real genes except for the inclination angle. The inclination is treated as an option in order to take into account the frozen-orbit constraint. If the orbit is circular, the angle can vary between 0 and 180. Otherwise, the angle is chosen between two values (63.4 or 116.6 degrees). The bound of the semimajor axis is between 4000 and 10000 km. The upper bound of the eccentricity is given by the condition that the orbit's periapsis is larger than 3500 km, which gives about 100 km altitude margin to the Mars radius (3396 km).

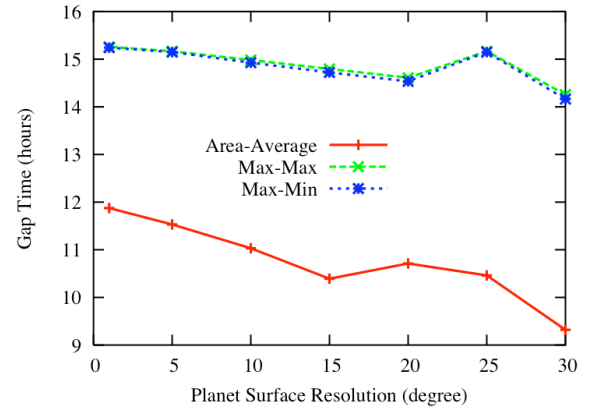


Figure 1. Telecommunication gap times with respect to the resolution of the planet surface.

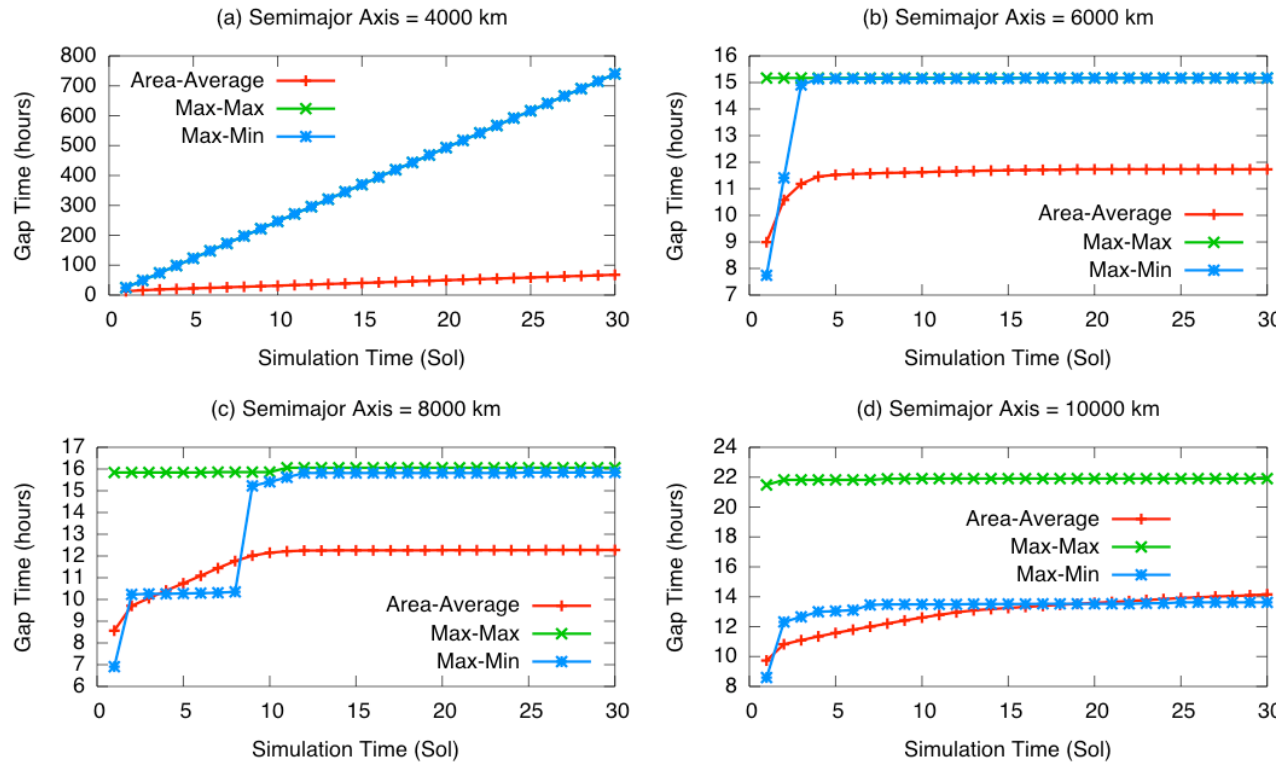


Figure 2. Telecommunication gap times with respect to simulation time for orbits with the semimajor axes of 4000, 6000, 8000, and 10000 km and with the inclination angle of 15 degrees. Except for the orbit with the semimajor axis of 4000 km, the telecommunication gap times converge as the simulation time approaches to 10-20 Sols.

Table 1. Optimal orbit solutions for three performance metrics, found by the genetic algorithm with TOAST tool. The three metrics are 1) area-weighted average gap time (Area-Average), 2) global maximum of local maximum gap time (Max-Max), 3) global maximum of local minimum gap time (Max-Min). The performance of the optimal solutions is compared with that of Mars Telecommunication Orbiters' candidate orbits.

Optimization metrics	Semimajor axis (km)	Eccentricity	Inclination (degree)	Argument of perigee (degree)	Gap Time (Hours)		
					Area-Average	Max-Max	Max-Min
Area-Average	6983.5	0	86.2	N/A	7.08	11.78	7.30
Max-Max	6090.3	0	89.8	N/A	7.52	9.86	9.85
Max-Min	6993.1	0	116.7	N/A	8.47	11.09	6.98
Mars Telecommunication Orbiter's Candidate Orbits*							
CSS4450N	7846.2	0	130.2	N/A	10.05	12.41	12.40
MACCI4N	8114.8	0.47	116.6	132.6	9.77	18.41	12.60
CCS4N	8118.0	0	136.7	N/A	8.23	12.90	12.26
ESS4N	8117.2	0.2	132.2	137.2	8.65	17.85	12.32

* There are 227 orbits that are identified and studied as candidates for the Mars Telecommunication Orbiter. Listed are four of the candidates in Ref. 7.

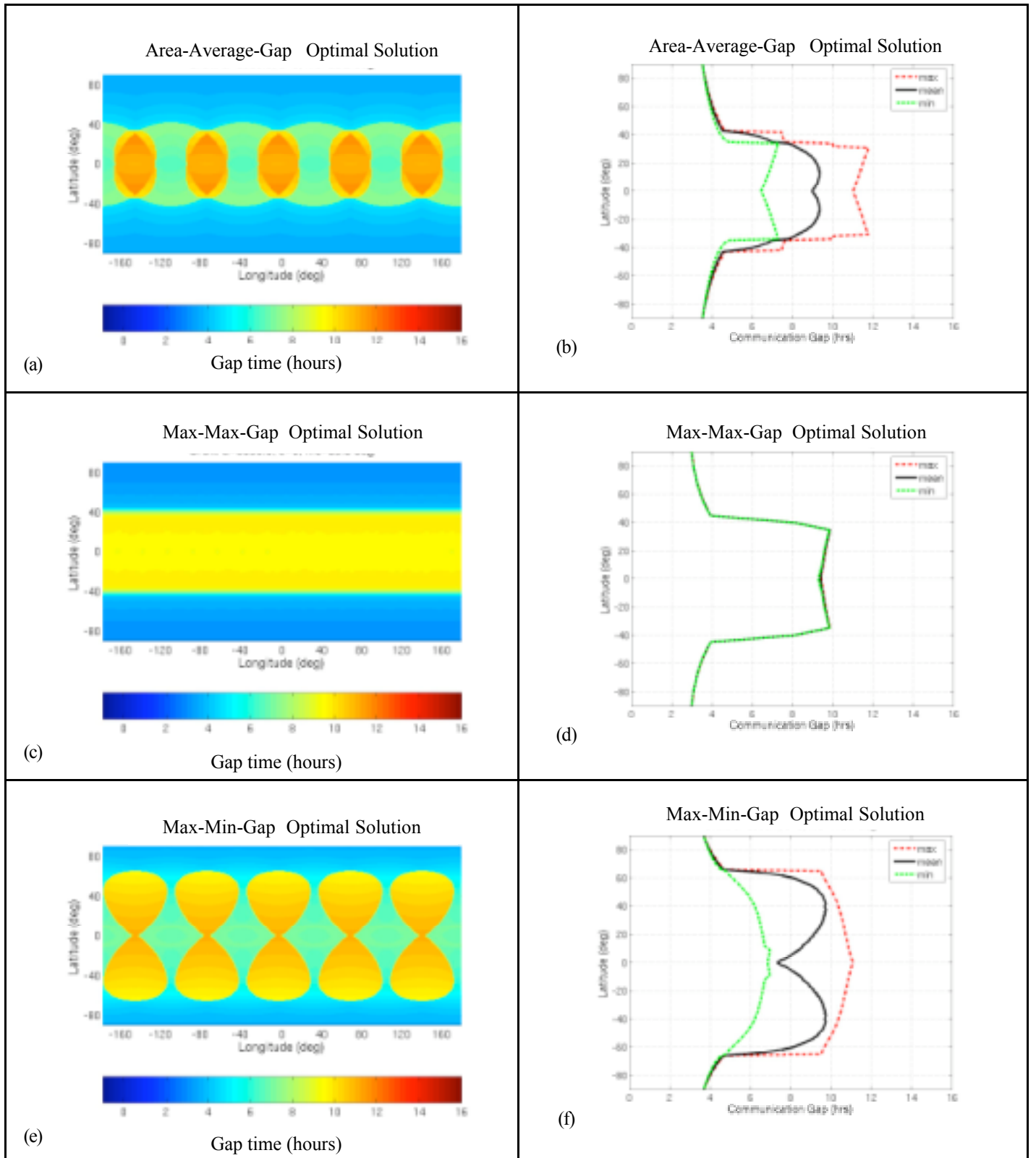


Figure 3. Global maps of the optimal solutions for each of the three performance metrics: (a,b) Area-Average, (c,d) Max-Max, and (e,f) Max-Min. The telecommunication gap times in unit of hours at each latitude and longitude grid are in (a,c,e). The global average, maximum, and minimum of the gap times over longitude at each latitude is given in (b,d,f).

Optimal Solutions

The optimal orbit solutions found by the genetic algorithm with TOAST are listed in Table 1. A different performance metric leads to a distinct solution. Common features among the three optimal solutions are that they are all circular orbits and highly inclined. With the optimal solutions, their global maps of the telecommunication gaps are plotted in Figure 3. Although the daily ground-track repeating constraint is not imposed, two of the optimal solutions (Area-Average and Max-Min) show the daily ground-track repeating feature. The orbit period is close to one fifth of a Martian Sol. The global maps were obtained with the simulation time of 15 Sols, and it shows that the local gap time is repeating during the 15 Sols. The Max-Max solution is a polar orbit and thus the gap time is uniform across the longitude axis.

Parametric Studies

We investigate why all the optimal solutions found are circular and highly inclined and to what extent the optimal solutions are better than other solutions. The dependences of the metric values on orbit variables are explored via two parametric studies. One parametric study is set up by incrementally changing the semimajor axis and inclination angles while the eccentricity is set to zero for a circular orbit. The other parametric study is set up by gradually changing the eccentricity and the argument of perigee while fixing the semimajor axis to 7000 km and choosing the inclination angle between 63.4 or 116.6 for an elliptical orbit. Note that a circular orbit does not have a restriction in choosing the inclination angle while an elliptical orbit does due to the frozen orbit requirement.

Figure 4 shows the dependences of the three metric values on semimajor axis and inclination angle for circular orbits. Particularly, Figure 4(a) shows the best metric value at a given semimajor axis across inclination angles for a circular orbit. The dependence indicates that the optimal semimajor axis is around 7000 km for the Area-Average and Max-Min metrics and around 6000 km for the Max-Max metric. The metric dependence on the semimajor axis is very spiky. This may be attributed to the limited resolution of the planet surface (5 degrees). The Area-Average metric is smoother than other metrics because it is the gap time averaged over the surface grids. Figure 4(b) shows the best metric value at a given inclination angle across semimajor axes for a circular orbit. The best inclination angle is around 90, 100, 115 degrees for Area-Average, Max-Max, Max-Min metrics, respectively.

Figure 5 shows the results of the parametric study for elliptical orbits. The best metric value at a given eccentricity across arguments of perigee with each of the two allowed inclination angles is plotted in Figure 5(a). Figure 5(b) shows the best metric value at a given argument of perigee across eccentricities. Generally, the inclination of 117 degrees yields better performance than that of 63 degrees. For a given inclination angle, the

performance improves (i.e. lower gap-time) as the eccentricity decreases. The dependence of the gap-time on the argument of perigee is marginal except for the Max-Min gap with the inclination of 117 degrees, where there is an abrupt change near 0 and 180 degrees. Comparison with MTO candidate solutions

Comparison with MTO Candidate Solutions

Finally, the performance of the found optimal solutions is compared with that of four candidate orbits of Mars Telecommunications Orbiter (MTO) [7]. Table 1 shows that our optimal solutions are better than the candidate solutions in terms of the three metrics we considered in the optimization process. It should be noted that the MTO candidate solutions have been identified by considering several different aspects that are not included in our optimization process. For example, all of the MTO candidate solutions are sun-synchronous. They also have high altitudes to obtain long pass durations and large foot-prints. Some of the candidate solutions (MACCI4N, CCS4N, ESS4N) have the feature of daily repeating ground track. In our optimization process, we did not impose the constraints for sun-synchronous, ground-track repeating, or a high altitude.

5. CONCLUSIONS

We have developed a method to select an optimal orbit solution for orbiters based on telecommunication performance metrics. The method uses TOAST (the Telecom Orbit Analysis and Simulation Tool) to estimate the performance metrics and uses a genetic algorithm to optimize the metrics. The developed method is applied to an orbit selection for general Mars telecommunication orbiters with the frozen orbit constraint. The optimization goal is to reduce the telecommunication gap time, and three relevant performance metrics are constructed: 1) area-weighted average gap time, 2) global maximum of local maximum gap time, 3) global maximum of local minimum gap time.

In order to ensure the reliable measurement of the performance metric, the convergence of the metric value is monitored with respect to the resolution of the planet surface and the simulation time. With the proper resolution and simulation time, an optimal solution for each metric is found. The common features among the optimal solutions are that they are all circular and highly inclined. Separate parametric studies are performed to investigate the dependence of the metric functions on orbit element variables. The parametric studies yield results that are consistent with the optimization results: a lower eccentricity and an inclination angle around 80-120 are preferred.

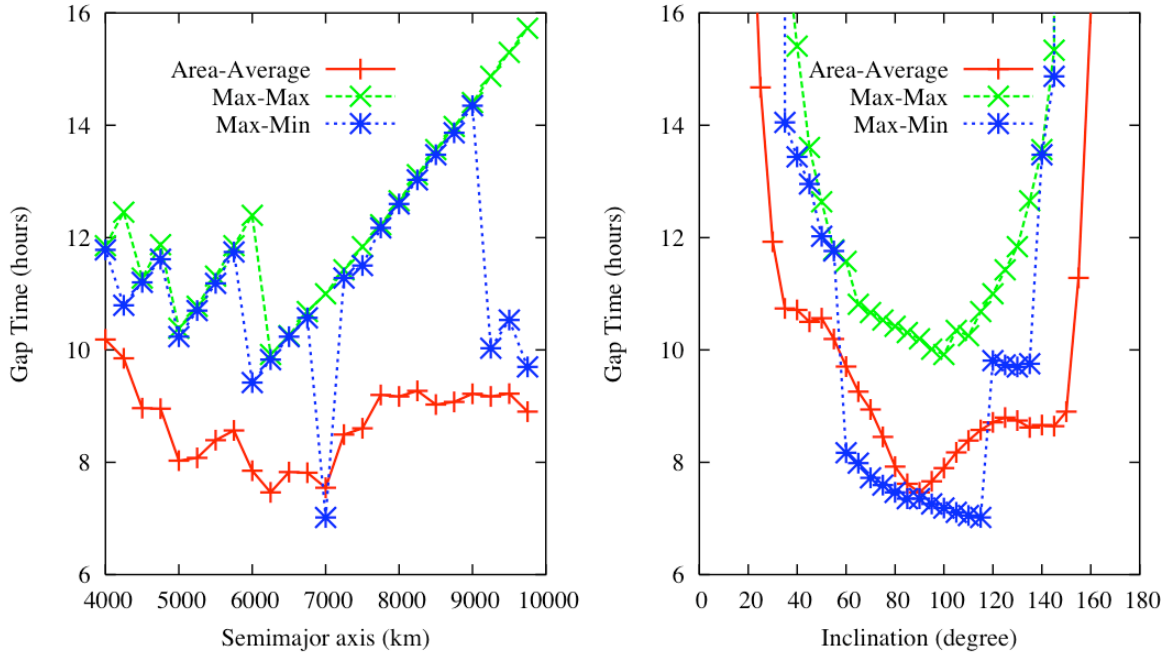


Figure 4. Parametric study for the dependences of three performance metrics (Area-Average, Max-Max, Max-Min gap time) on semimajor axis and inclination for circular orbits.

Gap time (hours)

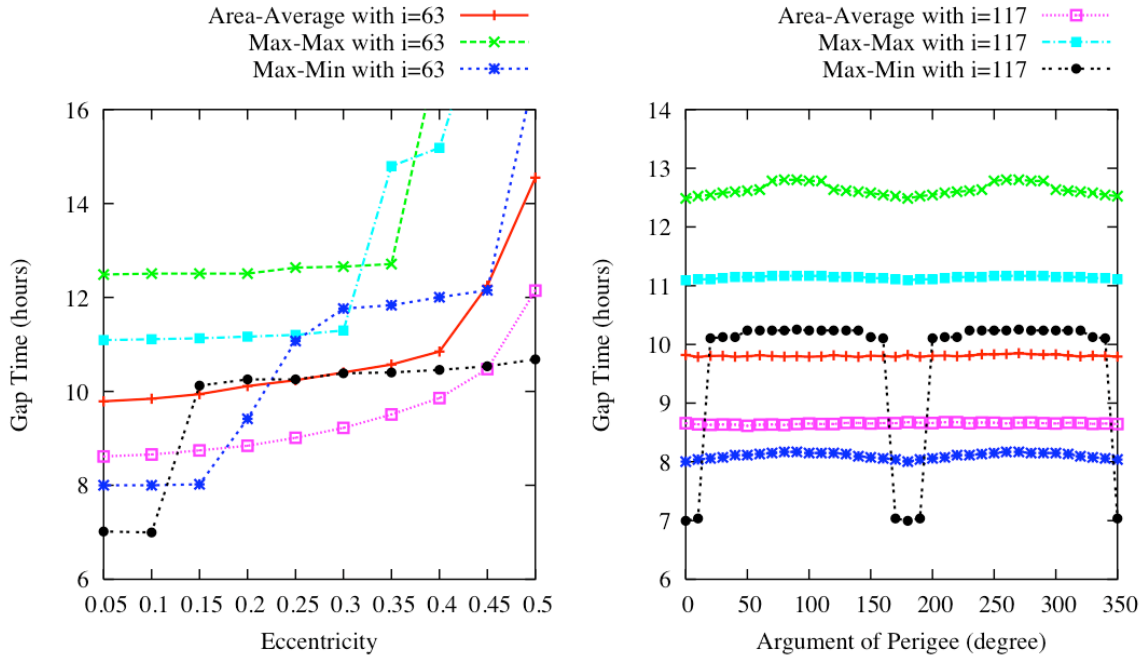


Figure 5. Parametric study for the dependences of three performance metrics on eccentricity and argument of perigee for elliptical orbits with two inclination angles (63 degrees or 117 degrees) allowed for frozen orbits.

The confirmed optimal solutions are compared with several candidate orbits for the Mars Telecommunication Orbiter (MTO). The MTO candidates are identified after considering different metrics/criteria and constraints. As a result, the MTO solutions are different from our solutions. This reveals complexity and trade-off in the orbit selection process, since there are many relevant metrics and constraints to consider and the prioritization of the metrics and constraints is mission-specific. Future work includes the assessment of a trade-off between different metrics and constraint/requirements.

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BIOGRAPHY



Seungwon Lee is a member of the technical staff at the Jet Propulsion Laboratory of California Institute of Technology. Her research interest includes evolutionary computing methods, Astrodynamics, materials and nanostructure simulation, and parallel cluster computing. She received her B.S. and M.S in Physics from the Seoul National University in 1995 and 1997, and her Ph.D. in Physics from the Ohio State University in 2002. Her work is documented in numerous journals and conference proceedings.



Charles H. Lee is an Associate Professor of Mathematics at the California State University Fullerton and a faculty part time staff in the Communications Systems Research Section (331) at the Jet Propulsion Laboratory. Before becoming a faculty member in 1999, he spent three years as a Post-Doctorate fellow at the Center for Research in Scientific Computation, Raleigh, North Carolina, where he was the recipient of the 1997-1999 National Science Foundation Industrial Post-Doctorate Fellowship. His research has been Computational Applied Mathematics with emphases in Control, Fluid Dynamics, Smart Material Structures and recently Telecommunications and Biomedical Engineering. He received his Doctor of Philosophy degree in Applied Mathematics in 1996 from the University of California at Irvine.



Charles D. Edwards, Jr. received his A.B. degree in Physics from Princeton University in 1979 and his Ph.D. in Physics from the California Institute of Technology in 1984. Since then he has worked at NASA's Jet Propulsion Laboratory, where he currently serves as Manager of the Mars Network Office and as Chief Telecommunications Engineer for the Mars Exploration Program, leading the development of a dedicated orbiting infrastructure at Mars providing essential telecommunications and navigation capability for future Mars exploration. Prior to that, he managed the Telecommunications and Mission Operations Technology Office, overseeing a broad program of research.



Kar-Ming Cheung is a Technical Group Supervisor in the Communications Systems Research Section at JPL. His group provides telecom analysis support and develops the operational telecom analysis and predict generation tools for current

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